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LETTER TO THE EDITOR

AC conductance, transfer matrix and small-frequency expansion in quasi-one-dimensional systems

J Cohen and Y Avishai

Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva, Israel

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Abstract. The AC conductance $G(E, \omega)$ (at Fermi energy E and frequency ω) of a quasi-one-dimensional system of finite length L governed by a tight-binding model Hamiltonian is expressed in terms of the pertinent transfer matrices. Consequently, one can study dynamic response functions of disordered systems within the powerful framework of random (transfer) matrices, which proved to be extremely useful in the analysis of DC conductance. We employ this formalism to investigate the low-frequency behaviour of $\ln G(E, \omega)$. As expected, the linear term vanishes, whereas the quadratic term can be expressed in terms of the eigenvalues of the DC transfer matrix. In a strictly one-dimensional case it can be written in terms of the DC conductance and its energy derivatives. The thermodynamic limit $L \to \infty$ cannot be taken term by term. It is conjectured that, in general, $\langle \ln G(E, \omega) \rangle$ is not self-averaging, in agreement with previous numerical results.

The use of the transfer matrix algorithm is a very important tool in the investigation of DC conductance of mesoscopic systems. Many physical concepts such as weak localization, positive magneto-conductance, universal conductance fluctuations and universal corrections to localization length are elegantly analysed in terms of random transfer matrix theory. In addition, the use of transfer matrix formulation proves to be very useful in actual numerical computation, especially for disordered systems. While much progress has been recorded in applying transfer matrix techniques to the study of DC conductance, it is still not in frequent use in the investigation of AC conductance. This is not surprising, since, in that case, it is not enough to know the total transfer matrix (denoted hereafter by $\mathbf{T}(E)$). Inspection of the Kubo formula shows that in order to evaluate the AC conductance, the wave functions and the current must be evaluated inside the conducting system. Yet this information is available anyway when the total transfer matrix is computed, since one has to know the intermediate transfer matrices that transform the wave function from one point to another within the system.

In one of our earlier works [1], we investigated the AC conductance $G(E, \omega)$ (at Fermi energy E and frequency ω) of a continuous quasi-one-dimensional system of finite length L. Within the standard linear response theory, a formalism was developed which uses the transfer matrices of the system in its computation. This formalism is based on the independent particle model, although the inclusion of electron–electron interaction within the screening approximation is, in principle, possible. Short- and long-wavelength spatial oscillations of the current response are included, but if the short period is eliminated by dephasing, the AC response reduces to an analogous expression derived recently [2].

The purpose of the present work is twofold. First, the formalism will be extended also for tight-binding models. As it turns out, the derivation of the AC conductance, starting

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from the Anderson tight-binding model Hamiltonian and the Kubo formula, requires some manipulations, since the transfer matrices are needed in the plane wave representation. We skip some technical details and present in (10) the dynamic conductance $G(E,\omega)$ in terms of the intermediate transfer matrices.

Our second goal is to study the low-frequency expansion of $\ln G(E,\omega)$ since it is related to the question of localization at finite frequency. We show that the first-order frequency correction vanishes (this is to be expected) and then obtain in (27) a closed expression for the quadratic term (under some reasonable approximations), in terms of the eigenvalues of the matrix $\mathbf{T}^{\dagger}\mathbf{T}$. We also address the question of whether $\ln G(E,\omega)$ is self-averaging. Indeed, in the DC case, and for $L \to \infty$, $-\ln G(E,0)/2L$ is independent of L. (For strictly one-dimensional systems, it is equal to $1/\xi(E)$, the inverse localization length.) At finite frequency, the limiting procedure turns out to be more difficult to perform. Taking the limit $L \to \infty$ on each term of the power series in ω proves to be ill-defined. We conjecture that, in general, $\ln G(E,\omega)$ is not self-averaging, confirming earlier numerical results.

Consider an electron hopping on a 2D square lattice (with lattice constant $a = 1$), of infinite length in the x direction, and width M in the y direction. The dynamics of the electron is governed by the tight-binding Hamiltonian [3,4]

$$H = \sum_{j=-\infty}^{+\infty} \left\{ \sum_{m=1}^{M} |jm\rangle \epsilon_{j,m} \langle jm| + \sum_{m=1}^{M-1} (|jm\rangle\langle jm+1| + |jm+1\rangle\langle jm|) + V \sum_{m=1}^{M} (|jm\rangle\langle j+1m| + |j+1m\rangle\langle jm|) \right\} \tag{1}$$

where $\{|jm\rangle\}$ is a complete set of orthonormal and localized states associated with the lattice sites (j,m). The hopping matrix element between nearest neighbours in the y direction is taken to be the unit of energy, while that between nearest neighbours in the x direction is V. We adopt hard wall boundary conditions in the y direction. The disorder is implemented as random fluctuations of the site energies within a finite portion of the entire system

$$\epsilon_{jm} = \begin{cases} 0 & j \leqslant 0,\ j > L \\ \epsilon_{jm} & 1 \leqslant j \leqslant L. \end{cases} \tag{2}$$

Starting with the Schrodinger equation $H\Psi = E\Psi$ the wave function is expanded in terms of lattice site states

$$|\Psi\rangle = \sum_{jm} \left[a_{jm} e^{ik_m j} = b_{jm} e^{-ik_m j} \right] |jm\rangle \tag{3}$$

where k_m and E are related through

$$E = 2V \cos k_m + E_m \tag{4}$$

in which $E_m = 2\cos[\pi m/(M+1)]$ $(m = 1, 2, \ldots, M)$ are the transverse energies of the standing waves. The $2M$ components (a_{jm}, b_{jm}) with $m = 1, 2, \ldots, M$ are denoted by the vector (a_j, b_j). The transformation from (a_{j-1}, b_{j-1}) to (a_j, b_j) is carried out by a transfer matrix in the plane wave representation, namely,

$$\begin{pmatrix} a_j \\ b_j \end{pmatrix} = \tau^{(j)} \begin{pmatrix} a_{j-1} \\ b_{j-1} \end{pmatrix} = \tau^{(j)} \tau^{(j-1)} \ldots \tau^{(1)} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \mathbf{T}^{(j)} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} \tag{5}$$

where

$$\tau^{(j)} = \mathbf{1} + \frac{\mathrm{i}}{2}\begin{pmatrix} \epsilon^{(j)} & 0 \\ 0 & \epsilon^{(j)} \end{pmatrix}\begin{pmatrix} \alpha & [\beta^{(j)}]^* \\ -\beta^{(j)} & -\alpha \end{pmatrix}. \tag{6}$$

Here $\mathbf{1}$ is the $2M \times 2M$ unit matrix, and the matrices $\epsilon^{(j)}$, α and $\beta^{(j)}$ are diagonal $M \times M$ matrices with elements

$$[\epsilon^{(j)}]_{mm} = \epsilon_{j,m} \qquad [\alpha]_{mm} = \frac{1}{\sin k_m} \qquad [\beta^{(j)}]_{mm} = \frac{\exp(2\mathrm{i}k_m j)}{\sin k_m}. \tag{7}$$

In (5), $\mathbf{T}^{(j)}$ is the transfer matrix in the plane wave representation. The transfer matrix through the entire system is $\mathbf{T} = \mathbf{T}^{(j=L)}$. Having obtained the transfer matrix in the plane wave representation we now use it to compute the AC conductance.

Following Fisher and Lee [5] we assume that a spatially homogeneous electric field $\boldsymbol{E} = \{E_0 \exp(\mathrm{i}\omega t), 0, 0\}$ is applied within a finite portion of the system of length L, located between two ideal leads. The AC conductance $G(E, \omega)$ can be defined by the energy absorption rate $P = G(E, \omega)(E_{\mathrm{rms}}L)^2$ where E_{rms} is the root of the mean square of $\boldsymbol{E}$. In the linear response approximation one obtains

$$G(E, \omega) = \frac{h}{2L^2}\int \mathrm{d}\epsilon \frac{f(\epsilon) - f(\epsilon + \hbar\omega)}{\hbar\omega} \sum_{ab} \left| \sum_{j=1}^{L} \langle a|J(j)|b\rangle \right|^2 \delta(\epsilon - E_a)\delta(\epsilon - E_b + \hbar\omega) \tag{8}$$

where $\langle a|, |b\rangle$ and E_a, E_b are eigenstates and eigenenergies of the entire system, and $f(\epsilon) = \{1 + \exp[(\epsilon - E)/kT]\}^{-1}$ is the Fermi distribution function. The current operator $J(j)$ is given by

$$J(j) = \frac{eV}{\mathrm{i}\hbar} \sum_{m=1}^{M} (|j+1, m\rangle\langle j, m| - |j, m\rangle\langle j+1, m|). \tag{9}$$

The evaluation of $G(E, \omega)$ in terms of transfer matrices is straightforward and follows the procedure explained in [1]. The following steps are to be carried out.

(i) Start from a given initial vector of coefficients (at $j = 0$) and use the expansion (3) to express the current (9) in terms of the transfer matrices $\mathbf{T}^{(j)}$. Anticipating the energy integration in (8), this procedure should be executed for two energies, $E_a = E$ and $E_b = E + \hbar\omega$.

(ii) Substitute the current computed in step (i) into (8) and perform all the integrals and sums.

(iii) Eliminate the dependence on the initial vector of coefficients by noting that the conductance can be written as a trace of a certain matrix. The final expression reads

$$G(E, \omega) = \frac{e^2}{h}\int \mathrm{d}\epsilon \left(\frac{f(\epsilon) - f(\epsilon + \hbar\omega)}{\hbar\omega}\right) g(\epsilon, \omega) \tag{10}$$

where

$$g(E, \omega) = \mathrm{Tr}\left\{\tfrac{1}{2}\left[\boldsymbol{\Theta}(E)\boldsymbol{\Omega}(E, E + \hbar\omega)\boldsymbol{\Theta}(E + \hbar\omega)\boldsymbol{\Omega}^{\dagger}(E, E + \hbar\omega)\right]\right\}. \tag{11}$$

The matrix $\Theta(E)$ can be expressed in terms of the total transfer matrix or alternatively in terms of the reflection amplitudes matrix $\mathbf{r}(E)$,

$$\Theta(E) = 2\left[\mathbf{T}^{\dagger}(E)\mathbf{T}(E) + \mathbf{I}\right]^{-1} = \begin{pmatrix} \mathbf{I} & \mathbf{r}^{\dagger}(E) \\ \mathbf{r}(E) & \mathbf{I} \end{pmatrix}. \tag{12}$$

The dynamic matrix Ω is

$$\Omega(E, E+\hbar\omega) = \sum_{j=1}^{L} \left[\mathbf{T}^{(j)}(E)\right]^{\dagger} \mathbf{H}^{(j)}(E, E+\hbar\omega)\mathbf{T}^{(j)}(E+\hbar\omega). \tag{13}$$

The transfer matrices $\mathbf{T}$ and $\mathbf{T}^{(j)}$ have been defined above in (5) and (6). The $2M \times 2M$ matrix $\mathbf{H}$ contains kinematic terms determined by the momenta $k_n(E)$ and $q_n(E) \equiv k_n(E+\hbar\omega)$. It is useful to define $K_n \equiv k_n(E) + k_n(E+\hbar\omega)$, and $Q_n \equiv k_n(E) - k_n(E+\hbar\omega)$ (see (4) for the relation between E and $k_n(E)$). Explicitly (omitting energy arguments), we get

$$\mathbf{H}^{(j)} = \begin{pmatrix} \mathbf{H}_1^{(j)} & \mathbf{H}_2^{(j)} \\ -\left[\mathbf{H}_2^{(j)}\right]^{*} & -\left[\mathbf{H}_1^{(j)}\right]^{*} \end{pmatrix} \tag{14}$$

with diagonal $M \times M$ blocks

$$\left[\mathbf{H}_1^{(j)}\right]_{nm} = \delta_{nm} \frac{\sin(K_n/2)}{L\sqrt{\sin k_n \sin q_n}} \exp[\mathrm{i}(j+\tfrac{1}{2})Q_n] \tag{15a}$$

$$\left[\mathbf{H}_2^{(j)}\right]_{nm} = \delta_{nm} \frac{\sin(Q_n/2)}{L\sqrt{\sin k_n \sin q_n}} \exp[-\mathrm{i}(j+\tfrac{1}{2})K_n]. \tag{15b}$$

Equations (10) and (11) with the auxiliary definitions (12)–(15) complete our first objective. It gives the AC conductance $G(E, \omega)$ (for systems governed by a tight-binding model Hamiltonian) in terms of the intermediate transfer matrices $\mathbf{T}^{(j)}$ and the kinematic matrices $\mathbf{H}^{(j)}$ at energies E and $E + \hbar\omega$. It has the simple and transparent structure of a DC conductance formula in which the physical processes of emission and absorption of a single photon are taking place. Note that the above formalism holds also for the continuous model [1].

As a check on the validity of our equations we show that in the zero-frequency limit $\omega \to 0$ the AC conductance $G(E, \omega)$ reduces to the denominator-free version of the Büttiker–Landauer DC conductance. Indeed, the transfer matrices $\mathbf{T}^{(j)}(E)$ satisfy the current conservation (unitarity) relation

$$\left[\mathbf{T}^{(j)}(E)\right]^{\dagger} \Sigma \mathbf{T}^{(j)}(E) = \Sigma \qquad \text{with} \qquad \Sigma = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix}. \tag{16}$$

Using this relation in (11) after performing the energy integration (10) we obtain $\Omega \to \Sigma$. Replacing Ω by Σ in (11) and $\mathbf{T}(E+\hbar\omega)$ by $\mathbf{T}$, we get the Pichard formula for the DC conductance [6]

$$G = \frac{e^2}{h} \mathrm{Tr}\left\{2\left[\mathbf{T}^{\dagger}\mathbf{T} + \left(\mathbf{T}^{\dagger}\mathbf{T}\right)^{-1} + 2\mathbf{I}\right]^{-1}\right\}. \tag{17}$$

We now move on to carry out the second part of our work and use our results to investigate the low-frequency behaviour of the AC conductance at zero temperature and link it to the

eigenvalues of the matrix $\mathbf{T}^{\dagger}\mathbf{T}$. To be more specific, the relevant dimensionless parameter that should be kept small in the low-frequency expansion is $\omega L/v$ (where v is the Fermi velocity). Let us expand the dimensionless AC conductance $\Gamma(E,\omega) \equiv (h/e^2)G(E,\omega)$ up to second order in ω. This requires some technical steps which lead to expression (27) below. The uninterested reader can join the discussion after (24).

Using the notation $\dot{g}(E) = [\partial g(E,\omega)/\partial\omega]_{\omega=0}$ and $g'_{\mathrm{DC}}(E) = [\partial g(E,\omega)/\partial E]_{\omega=0}$ we have

$$\Gamma(E,\omega) = g_{\mathrm{DC}}(E) + \omega\left[\dot{g}(E) - \frac{\hbar}{2}g'_{\mathrm{DC}}(E)\right] + \frac{\omega^2}{2}\left[\ddot{g}(E) - \frac{\hbar^2}{6}g''_{\mathrm{DC}}(E)\right] \tag{18}$$

where $g_{\mathrm{DC}}(E)$ is the DC conductance, which can be written in terms of the matrix $\Theta(E)$ defined above (equation (12))

$$g_{\mathrm{DC}}(E) = \tfrac{1}{2}\mathrm{Tr}\,[\Theta(E)\Sigma\Theta(E)\Sigma]. \tag{19}$$

For the derivative of $g(E,\omega)$ at $\omega = 0$ one obtains

$$\left.\frac{\partial g(E,\omega)}{\partial\omega}\right|_{\omega=0} = \frac{1}{2}\left\{\mathrm{Tr}\left[\Theta(E)\Sigma\Theta(E)\left(\frac{\partial\Omega^{\dagger}}{\partial\omega} + \frac{\partial\Omega}{\partial\omega}\right)_{\omega=0}\right] + \mathrm{Tr}\left[\Theta(E)\Sigma\left(\frac{\partial\Theta(E+\hbar\omega)}{\partial\omega}\right)_{\omega=0}\Sigma\right]\right\}. \tag{20}$$

Using the relation $\mathrm{Tr}(\mathbf{AB}) = \mathrm{Tr}(\mathbf{BA})$ and some other simple algebraic manipulations, it is easily seen that the first term vanishes while the second term is

$$\mathrm{Tr}\left\{\Theta(E)\Sigma\left[\frac{\partial\Theta(E+\hbar\omega)}{\partial\omega}\right]_{\omega=0}\Sigma\right\} = \left[\frac{\partial g_{\mathrm{DC}}(E+\hbar\omega)}{\partial\omega}\right]_{\omega=0} = \hbar\frac{\mathrm{d}g_{\mathrm{DC}}(E)}{\mathrm{d}E}. \tag{21}$$

Substituting (20) and (21) into (18) shows that the first-order term in the frequency expansion vanishes, and hence

$$\Gamma(E,\omega) = g_{\mathrm{DC}}(E) + \frac{\omega^2}{2}\left[\ddot{g}(E) - \frac{\hbar^2}{6}g''_{\mathrm{DC}}(E)\right]. \tag{22}$$

To proceed further, we follow Büttiker (1992a) and Büttiker and Thomas (1992) [2] and use a couple of reasonable approximations which lead to a simplified form of $g(E,\omega)$. The short-wavelength spatial oscillation with period K^{-1} can be replaced by $2k(E)$ over a substantial range of frequencies. Furthermore, the long-wavelength period of oscillations Q^{-1} is replaced by v/ω, where v is the Fermi velocity. Under these two assumptions we can approximate the product of matrices $[\tau^{(J)}(E+\hbar\omega)]^{\dagger}\Sigma\tau^{(J)}(E) \simeq \Sigma$ (see (5) for the definition of $\tau^{(J)}$). Hence, we get

$$g(E,\omega) = \mathrm{Tr}\left\{\tfrac{1}{2}[\Theta(E)\Sigma\Theta(E+\hbar\omega)\Sigma]\right\} \tag{23}$$

where the matrices Θ and Σ have been defined in (12) and (16) respectively. In this representation we can employ (21) to express the second-order frequency derivative at zero frequency in terms of the second-order energy derivative

$$\left.\frac{\partial^2 g(E,\omega)}{\partial\omega^2}\right|_{\omega=0} = \frac{\hbar^2}{2}\mathrm{Tr}\left\{\Theta(E)\Sigma\frac{\mathrm{d}^2\Theta(E)}{\mathrm{d}E^2}\Sigma\right\} = \frac{\hbar^2}{2}\left\{\frac{\mathrm{d}^2 g_{\mathrm{DC}}(E)}{\mathrm{d}E^2} - \mathrm{Tr}\left[\frac{\mathrm{d}[\Theta(E)\Sigma]}{\mathrm{d}E}\frac{\mathrm{d}[\Theta(E)\Sigma]}{\mathrm{d}E}\right]\right\}. \tag{24}$$

Consider now the matrix [7]

$$\Lambda^{2L}(E) = \mathbf{T}^{\dagger}(E)\mathbf{T}(E) \tag{25}$$

with eigenvalues $\exp(\pm 2L\alpha_i)$. The DC conductance can be expressed as [8]

$$g_{\mathrm{DC}} = \sum_i \frac{1}{\cosh(2L\alpha_i) + 1} = \frac{1}{2}\sum_i \lambda_i^2 \tag{26}$$

where $\lambda_i = 1/\cosh(L\alpha_i)$ are the eigenvalues of the matrix multiplication $\Theta(E)\Sigma$. Using this expression we can obtain the second-order frequency correction to the steady state conductance, in terms of λ_i

$$\Gamma(E,\omega) = g_{\mathrm{DC}}(E) + \frac{(\hbar\omega)^2}{2}\left\{\frac{1}{3}g''_{\mathrm{DC}}(E) - \sum_i [\lambda_i'(E)]^2\right\}. \tag{27}$$

This result is general for quasi-one-dimensional systems with an arbitrary number M of channels. Its practical content cannot be overlooked, since, in this approximation, we do not have to know the exact frequency dependence of the $g(E,\omega)$. In other words, the AC conductance is determined through the energy dependence of the eigenvalues of $\mathbf{T}^{\dagger}\mathbf{T}$.

For strictly one-dimensional systems ($M = 1$), further progress can be made, which can shed light on the question of localization at finite frequency. In this case, the second-order correction can be written in terms of the derivative of $\ln g_{\mathrm{DC}}(E)$

$$\begin{aligned}\Gamma(\omega) &= g_{\mathrm{DC}}(E)\left\{1 + \frac{(\hbar\omega)^2}{6}\left[\frac{\mathrm{d}^2 \ln g_{\mathrm{DC}}(E)}{\mathrm{d}E^2} - \frac{1}{2}\left(\frac{\mathrm{d}\ln g_{\mathrm{DC}}(E)}{\mathrm{d}E}\right)^2\right]\right\} \\ &= g_{\mathrm{DC}}(E)\left[1 + \frac{(\hbar\omega)^2}{6}\gamma(E)\right]\end{aligned} \tag{28}$$

with the proviso that $\omega L/v$ is small, or equivalently,

$$\frac{(\hbar\omega)^2}{6}\gamma(E) < 1. \tag{29}$$

In the insulating regime where $L \gg \xi(E)$ (here $\xi(E)$ is the localization length at energy E), we may take $g_{\mathrm{DC}} \propto \exp(-2L/\xi)$. Then, to second order in the frequency ω we have

$$-\frac{\ln\Gamma(E,\omega)}{2L} = -\frac{\ln g_{\mathrm{DC}}(E)}{2L} - \frac{(\hbar\omega)^2}{6}\frac{\gamma(E)}{2L} = \frac{1}{\xi(E)} - \frac{(\hbar\omega)^2}{6}\frac{\gamma(E)}{2L}. \tag{30}$$

It is tempting at this point to take the thermodynamic limit $L \to \infty$ and to regard the limit of the right-hand side of (30) (if it exists) as the inverse localization length at finite frequency. Unfortunately, this is not possible in general, since $\gamma(E)$ is quadratic in L, and for large L and fixed frequency, the restriction (29) will no longer hold. Since the dynamic conductance depends on both frequency and length, the various limits ($L \to \infty$, $\omega \to 0$) have to be taken carefully. We have already asserted that as $\omega \to 0$, the DC conductance is recovered, so that the procedure $\lim_{L\to\infty}\lim_{\omega\to 0}(\ln\Gamma(E,\omega)/2L) = 1/\xi(E)$ is well defined. On the other hand, if one needs to keep the frequency small but finite, and then to go to the thermodynamic limit, the procedure $L \to \infty$ should be carried out starting from (10) and (11) directly without relying on low-frequency expansion. In that case, our numerical

calculations (which are not yet completed) indicate that $-\ln\Gamma(E,\omega)$ is proportional to $\ln L$ (except for some special energies), in agreement with earlier numerical results (see Mašek and Kramer 1988 [3]). Therefore we must conclude that $\ln\Gamma(E,\omega)$ is not self-averaging, and hence the concept of localization at finite frequency is ill-defined, but still, the AC conductance tends to zero as $L\to\infty$. Somewhat unexpectedly, this later result seems to violate Mott's law (asserting finite bulk AC conductivity $\sigma(\omega)$ in the insulating regime). This point has already been discussed elsewhere [3], where it has been conjectured that the measured conductance cannot be simply related to the conductivity by Ohm's law.

Finally, let us apply our results on two one-dimensional disordered systems in which the energy dependence of the localization length $\xi(E)$ is known analytically (albeit approximately). The first one is in the Anderson model, where the random site energies ϵ_j are uniformly distributed within the interval $[-W/2, W/2]$. In the weak-disorder limit $W\to 0$ and for energies not too close to the band edges ($E\neq\pm 2$), the standard non-degenerate perturbation theory for the localization length yields [9] $\xi(E)\simeq(4-E^2)/W^2$ (the hopping matrix element V is taken to be unity). In the band centre ($E=0$) we then obtain

$$-\frac{\ln\Gamma(E=0,\omega)}{2L}=\frac{1}{\xi(E=0)}\left[1+\frac{1}{3\xi(E=0)}\frac{(\hbar\omega)^2}{W^2}\right]. \tag{31}$$

In this particular case, the right-hand side is independent of L, and $\ln\Gamma(E=0,\omega)$ is self-averaging (this is true up to ω^2). The right-hand side of (31) can therefore be regarded as the inverse localization length at finite frequency. We must emphasize here that this result is special and may not be valid away from the band centre.

As another example we consider a particle moving in a one-dimensional Gaussian white noise potential $U(x)$, with $\langle U(x)\rangle=0$ and $\langle U(x)U(x')\rangle=2D\delta(x-x')$. The localization length is given by [10],

$$\xi(E)\simeq\frac{2E}{D}\qquad E\to+\infty \tag{32}$$

which implies

$$-\frac{\ln\Gamma(\omega)}{2L}\simeq\frac{1}{\xi}\left\{1+\frac{1}{3}\left(\frac{\hbar\omega}{E}\right)^2\left[1+\frac{L}{2\xi}\right]\right\}. \tag{33}$$

Here we see that the correction depends on L, and that taking the limit $L\to\infty$ on the right-hand side is meaningless. This is not surprising, since we must always respect the inequality (29).

To conclude, in the present work we have achieved two main goals. (a) We have developed a formal algorithm for the computation of dynamic response functions of a quasi-one-dimensional mesoscopic system in terms of its pertinent transfer matrices (in the plane wave representation), starting from a tight-binding Hamiltonian and the Kubo formula. (b) We investigated the low-frequency behaviour of the conductance. As long as the length L of the system is kept finite, an expansion in a power series of the frequency ω is well defined. As expected for a dissipative conductance, the first-order term vanishes (in fact, this is expected for any odd power). The correction then starts at second order in the frequency and can be related to the energy derivatives of the DC conductance. In the thermodynamic limit, the low-frequency expansion of the log of the conductance is not useful. Apparently, this quantity is not self-averaging.

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